NUMBER THEORY AND DIOPHANTINE ANALYSIS.

203. Proposed by R. D. CARMICHAEL, Indiana University.

Find solutions in integers of the equation $2x^2 + 1 = 3y^2$.

204. Proposed by E. T. BELL, New York City.

Show that a necessary and sufficient condition that $6n + 1$ be a prime number is that no one of the quantities $(3n - r)/(2r + 1)$ for $r = 1, 2, 3, \cdots n - 1$ be an integer; similarly for $6n - 1$, the quantities being $(3n - r)/(2r - 1)$ for $r = 2, 3, 4, \cdots n$.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

389. Proposed by W. W. BEMAN, University of Michigan.

If $e^{e^x} = 1 + a_1x + a_2x^2 + a_3x^3 + \cdots$, prove that

$$na_n = \sum_{k=1}^{k=n} \frac{1}{(k-1)!} a_{n-k} \quad \text{or} \quad n!\, a_n = \sum_{k=1}^{k=n} \frac{(n-1)!}{(k-1)!} a_{n-k}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

We shall assume that the series

$$e^{e^x} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

may be differentiated term by term. We then obtain

$$e^x \cdot e^{e^x} = a_1 + 2a_2x + 3a_3x^2 + \cdots + na_nx^{n-1} + \cdots$$

or

$$\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \cdots\right)\left(a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + \cdots\right)$$
$$= a_1 + 2a_2x + 3a_3x^2 + \cdots + na_nx^{n-1} + \cdots.$$

Multiply the two series and equate coefficients of like powers of x. We then have

$$\begin{aligned}
a_1 &= a_0,\\
2a_2 &= a_1 + \frac{a_0}{1!},\\
3a_3 &= a_2 + \frac{a_1}{1!} + \frac{a_0}{2!},\\
4a_4 &= a_3 + \frac{a_2}{1!} + \frac{a_1}{2!} + \frac{a_0}{3!},\\
\cdots &= \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
na_n &= a_{n-1} + \frac{a_{n-2}}{1!} + \frac{a_{n-3}}{2!} + \frac{a_{n-4}}{3!} + \cdots \frac{a_0}{(n-1)!}\\
&= \sum_{k=1}^{k=n} \frac{1}{(k-1)!} a_{n-k}.
\end{aligned}$$

Solved similarly by A. L. MCCARTY and ELMER SCHUYLER.